

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

- 22.1 Heat Engines and the Second Law of Thermodynamics
- 22.2 Heat Pumps and Refrigerators
- 22.3 Reversible and Irreversible Processes
- 22.4 The Carnot Engine
- 22.5 Gasoline and Diesel Engines
- 22.6 Entropy
- 22.7 Entropy Changes in Irreversible Processes
- 22.8 Entropy on a Microscopic Scale

ANSWERS TO QUESTIONS

- Q22.1** First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.
- Q22.2** It is easier to control the temperature of a hot reservoir. If it cools down, then heat can be added through some external means, like an exothermic reaction. If it gets too hot, then heat can be allowed to “escape” into the atmosphere. To maintain the temperature of a cold reservoir, one must remove heat if the reservoir gets too hot. Doing this requires either an “even colder” reservoir, which you also must maintain, or an endothermic process.
- Q22.3** A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c , and steam at T_h , the efficiency of the power plant goes as $\frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$ and is maximized for a high T_h .
- Q22.4** No. Any heat engine takes in energy by heat and must also put out energy by heat. The energy that is dumped as exhaust into the low-temperature sink will always be thermal pollution in the outside environment. So-called ‘steady growth’ in human energy use cannot continue.
- Q22.5** No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{eng} + |Q_c|$, and the second law implies $|Q_c| > 0$.
- Q22.6** Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. Clearly much of the energy goes into heat, which, through the cooling system, is dissipated into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into heat. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless heat in the atmosphere.

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- Q22.7** Suppose the ambient temperature is 20°C. A gas can be heated to the temperature of the bottom of the pond, and allowed to cool as it blows through a turbine. The Carnot efficiency of such an engine is about $e_c = \frac{\Delta T}{T_h} = \frac{80}{373} = 22\%$.
- Q22.8** No, because the work done to run the heat pump represents energy transferred into the house by heat.
- Q22.9** A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. The free flight of a projectile is nearly reversible.
- Q22.10** Below the frost line, the winter temperature is much higher than the air or surface temperature. The earth is a huge reservoir of internal energy, but digging a lot of deep trenches is much more expensive than setting a heat-exchanger out on a concrete pad. A heat pump can have a much higher coefficient of performance when it is transferring energy by heat between reservoirs at close to the same temperature.
- Q22.11** (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.
- (b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.
- Q22.12** Energy flows by heat from a hot bowl of chili into the cooler surrounding air. Heat lost by the hot stuff is equal to heat gained by the cold stuff, but the entropy decrease of the hot stuff is less than the entropy increase of the cold stuff. As you inflate a soft car tire at a service station, air from a tank at high pressure expands to fill a larger volume. That air increases in entropy and the surrounding atmosphere undergoes no significant entropy change. The brakes of your car get warm as you come to a stop. The shoes and drums increase in entropy and nothing loses energy by heat, so nothing decreases in entropy.
- Q22.13** (a) For an expanding ideal gas at constant temperature, $\Delta S = \frac{\Delta Q}{T} = nR \ln \left(\frac{V_2}{V_1} \right)$.
- (b) For a reversible adiabatic expansion $\Delta Q = 0$, and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for ΔS up to the value given in part (a).
- Q22.14** The rest of the Universe must have an entropy change of +8.0 J/K, or more.
- Q22.15** Even at essentially constant temperature, energy must flow by heat out of the solidifying sugar into the surroundings, to raise the entropy of the environment. The water molecules become less ordered as they leave the liquid in the container to mix into the whole atmosphere and hydrosphere. Thus the entropy of the surroundings increases, and the second law describes the situation correctly.

- Q22.16** To increase its entropy, raise its temperature. To decrease its entropy, lower its temperature. "Remove energy from it by heat" is not such a good answer, for if you hammer on it or rub it with a blunt file and at the same time remove energy from it by heat into a constant temperature bath, its entropy can stay constant.
- Q22.17** An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the organized motion of the falling water turns into disorganized molecular motion. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds organized cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy crashing down to disorder. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe. Your roommate's exercise puts energy into the room by heat.
- Q22.18** (a) Entropy increases as the yeast dies and as energy is transferred from the hot oven into the originally cooler dough and then from the hot bread into the surrounding air.
- (b) Entropy increases some more as you metabolize the starches, converting chemical energy into internal energy.
- Q22.19** Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy which it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere. The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed.
- Q22.20** The statement is not true. Although the probability is not exactly zero that this will happen, the probability of the concentration of air in one corner of the room is very nearly zero. If some billions of molecules are heading toward that corner just now, other billions are heading away from the corner in their random motion. Spontaneous compression of the air would violate the second law of thermodynamics. It would be a spontaneous departure from thermal and mechanical equilibrium.
- Q22.21** Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. The accumulation of larger candies on top and smaller ones on the bottom implies a small increase in order, a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a bit of gravitational energy loss as the beans settle compactly together.

SOLUTIONS TO PROBLEMS

Section 22.1 Heat Engines and the Second Law of Thermodynamics

P22.1 (a) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{25.0 \text{ J}}{360 \text{ J}} = \boxed{0.0694} \text{ or } \boxed{6.94\%}$

(b) $|Q_c| = |Q_h| - W_{\text{eng}} = 360 \text{ J} - 25.0 \text{ J} = \boxed{335 \text{ J}}$

P22.2 $W_{\text{eng}} = |Q_h| - |Q_c| = 200 \text{ J}$ (1)

$e = \frac{W_{\text{eng}}}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} = 0.300$ (2)

From (2), $|Q_c| = 0.700|Q_h|$ (3)

Solving (3) and (1) simultaneously,

we have

(a) $|Q_h| = 667 \text{ J}$ and

(b) $|Q_c| = 467 \text{ J}$.

P22.3 (a) We have $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} = 0.250$

with $|Q_c| = 8000 \text{ J}$, we have $|Q_h| = \boxed{10.7 \text{ kJ}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = 2667 \text{ J}$

and from $\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t = \frac{W_{\text{eng}}}{\mathcal{P}} = \frac{2667 \text{ J}}{5000 \text{ J/s}} = \boxed{0.533 \text{ s}}$.

***P22.4** We have $Q_{hx} = 4Q_{hy}$, $W_{\text{eng}x} = 2W_{\text{eng}y}$ and $Q_{cx} = 7Q_{cy}$. As well as $Q_{hx} = W_{\text{eng}x} + Q_{cx}$ and $Q_{hy} = W_{\text{eng}y} + Q_{cy}$. Substituting, $4Q_{hy} = 2W_{\text{eng}y} + 7Q_{cy}$

$4Q_{hy} = 2W_{\text{eng}y} + 7Q_{hy} - 7W_{\text{eng}y}$

$5W_{\text{eng}y} = 3Q_{hy}$

(b) $e_y = \frac{W_{\text{eng}y}}{Q_{hy}} = \frac{3}{5} = \boxed{60.0\%}$

(a) $e_x = \frac{W_{\text{eng}x}}{Q_{hx}} = \frac{2W_{\text{eng}y}}{4Q_{hy}} = \frac{2}{4}(0.600) = 0.300 = \boxed{30.0\%}$

***P22.5** (a) The input energy each hour is

$$(7.89 \times 10^3 \text{ J/revolution})(2500 \text{ rev/min}) \frac{60 \text{ min}}{1 \text{ h}} = 1.18 \times 10^9 \text{ J/h}$$

$$\text{implying fuel input } (1.18 \times 10^9 \text{ J/h}) \left(\frac{1 \text{ L}}{4.03 \times 10^7 \text{ J}} \right) = \boxed{29.4 \text{ L/h}}$$

(b) $Q_h = W_{\text{eng}} + |Q_c|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t} + \frac{|Q_c|}{\Delta t}$$

$$\begin{aligned} \text{Useful power output} &= \frac{W_{\text{eng}}}{\Delta t} = \frac{Q_h}{\Delta t} - \frac{|Q_c|}{\Delta t} \\ &= \left(\frac{7.89 \times 10^3 \text{ J}}{\text{revolution}} - \frac{4.58 \times 10^3 \text{ J}}{\text{revolution}} \right) \frac{2500 \text{ rev}}{1 \text{ min}} \frac{1 \text{ min}}{60 \text{ s}} = 1.38 \times 10^5 \text{ W} \end{aligned}$$

$$\mathcal{P}_{\text{eng}} = 1.38 \times 10^5 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = \boxed{185 \text{ hp}}$$

$$(c) \quad \mathcal{P}_{\text{eng}} = \tau \omega \Rightarrow \tau = \frac{\mathcal{P}_{\text{eng}}}{\omega} = \frac{1.38 \times 10^5 \text{ J/s}}{(2500 \text{ rev}/60 \text{ s}) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right)} = \boxed{527 \text{ N} \cdot \text{m}}$$

$$(d) \quad \frac{|Q_c|}{\Delta t} = \frac{4.58 \times 10^3 \text{ J}}{\text{revolution}} \left(\frac{2500 \text{ rev}}{60 \text{ s}} \right) = \boxed{1.91 \times 10^5 \text{ W}}$$

P22.6 The heat to melt 15.0 g of Hg is $|Q_c| = mL_f = (15 \times 10^{-3} \text{ kg})(1.18 \times 10^4 \text{ J/kg}) = 177 \text{ J}$

The energy absorbed to freeze 1.00 g of aluminum is

$$|Q_h| = mL_f = (10^{-3} \text{ kg})(3.97 \times 10^5 \text{ J/kg}) = 397 \text{ J}$$

and the work output is

$$W_{\text{eng}} = |Q_h| - |Q_c| = 220 \text{ J}$$

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{220 \text{ J}}{397 \text{ J}} = 0.554, \text{ or } \boxed{55.4\%}$$

The theoretical (Carnot) efficiency is $\frac{T_h - T_c}{T_h} = \frac{933 \text{ K} - 243.1 \text{ K}}{933 \text{ K}} = 0.749 = 74.9\%$

Section 22.2 Heat Pumps and Refrigerators

P22.7 $\text{COP}(\text{refrigerator}) = \frac{Q_c}{W}$

(a) If $Q_c = 120 \text{ J}$ and $\text{COP} = 5.00$, then $\boxed{W = 24.0 \text{ J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h = Q_c + W = 120 \text{ J} + 24 \text{ J} = \boxed{144 \text{ J}}$$

P22.8 $\text{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_c}{t} = (0.0300 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(22.0^\circ\text{C}) + (0.0300 \text{ kg})(3.33 \times 10^5 \text{ J/kg}) \\ + (0.0300 \text{ kg})(2090 \text{ J/kg}^\circ\text{C})(20.0^\circ\text{C}) = 1.40 \times 10^4 \text{ J/min}$$

or, $\frac{Q_c}{t} = 233 \text{ J/s}$.

Thus, the work done per sec = $\mathcal{P} = \frac{233 \text{ J/s}}{3.00} = \boxed{77.8 \text{ W}}$.

P22.9 (a) $\left(10.0 \frac{\text{Btu}}{\text{h} \cdot \text{W}}\right) \left(\frac{1055 \text{ J}}{1 \text{ Btu}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{1 \text{ W}}{1 \text{ J/s}}\right) = \boxed{2.93}$

(b) Coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5, $5 \frac{\text{Btu}}{\text{h} \cdot \text{W}} = \frac{10000 \text{ Btu/h}}{\mathcal{P}}$: $\mathcal{P} = \frac{10000 \text{ Btu/h}}{\frac{5 \text{ Btu}}{\text{h} \cdot \text{W}}} = 2000 \text{ W} = 2.00 \text{ kW}$

Energy purchased is

$$\mathcal{P} \Delta t = (2.00 \text{ kW})(1500 \text{ h}) = 3.00 \times 10^3 \text{ kWh}$$

$$\text{Cost} = (3.00 \times 10^3 \text{ kWh})(0.100 \text{ \$/kWh}) = \$300$$

With EER 10, $10 \frac{\text{Btu}}{\text{h} \cdot \text{W}} = \frac{10000 \text{ Btu/h}}{\mathcal{P}}$:

$$\mathcal{P} = \frac{10000 \text{ Btu/h}}{\frac{10 \text{ Btu}}{\text{h} \cdot \text{W}}} = 1000 \text{ W} = 1.00 \text{ kW}$$

Energy purchased is

$$\mathcal{P} \Delta t = (1.00 \text{ kW})(1500 \text{ h}) = 1.50 \times 10^3 \text{ kWh}$$

$$\text{Cost} = (1.50 \times 10^3 \text{ kWh})(0.100 \text{ \$/kWh}) = \$150$$

Thus, the cost for air conditioning is

$$\boxed{\text{half as much with EER 10}}$$

Section 22.3 Reversible and Irreversible Processes

No problems in this section

Section 22.4 The Carnot Engine

P22.10 When $e = e_c$, $1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|}$ and $\frac{W_{\text{eng}}}{\Delta t} = 1 - \frac{T_c}{T_h}$

(a) $|Q_h| = \frac{\left(\frac{W_{\text{eng}}}{\Delta t}\right) \Delta t}{1 - \frac{T_c}{T_h}} = \frac{(1.50 \times 10^5 \text{ W})(3600 \text{ s})}{1 - \frac{293}{773}}$

$$|Q_h| = 8.69 \times 10^8 \text{ J} = \boxed{869 \text{ MJ}}$$

(b) $|Q_c| = |Q_h| - \left(\frac{W_{\text{eng}}}{\Delta t}\right) \Delta t = 8.69 \times 10^8 - (1.50 \times 10^5)(3600) = 3.30 \times 10^8 \text{ J} = \boxed{330 \text{ MJ}}$

P22.11 $T_c = 703 \text{ K}$ $T_h = 2143 \text{ K}$

(a) $e_c = \frac{\Delta T}{T_h} = \frac{1440}{2143} = \boxed{67.2\%}$

(b) $|Q_h| = 1.40 \times 10^5 \text{ J}$, $W_{\text{eng}} = 0.420|Q_h|$
 $\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88 \times 10^4 \text{ J}}{1 \text{ s}} = \boxed{58.8 \text{ kW}}$

P22.12 The Carnot efficiency of the engine is $e_c = \frac{\Delta T}{T_h} = \frac{120 \text{ K}}{473 \text{ K}} = 0.253$

At 20.0% of this maximum efficiency, $e = 0.200(0.253) = 0.0506$

From the definition of efficiency $W_{\text{eng}} = |Q_h|e$

and $|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{10.0 \text{ kJ}}{0.0506} = \boxed{197 \text{ kJ}}$

P22.13 Isothermal expansion at $T_h = 523 \text{ K}$

Isothermal compression at $T_c = 323 \text{ K}$

Gas absorbs 1200 J during expansion.

(a) $|Q_c| = |Q_h| \left(\frac{T_c}{T_h} \right) = 1200 \text{ J} \left(\frac{323}{523} \right) = \boxed{741 \text{ J}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = (1200 - 741) \text{ J} = \boxed{459 \text{ J}}$

P22.14 We use $e_c = 1 - \frac{T_c}{T_h}$

as $0.300 = 1 - \frac{573 \text{ K}}{T_h}$

From which, $T_h = 819 \text{ K} = \boxed{546^\circ\text{C}}$

***P22.15** The efficiency is $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{|Q_c|}{|Q_h|}$

Then $\frac{T_c}{T_h} = \frac{\frac{|Q_c|}{\Delta t}}{\frac{|Q_h|}{\Delta t}}$

$$\frac{|Q_h|}{\Delta t} = \frac{|Q_c|}{\Delta t} \frac{T_h}{T_c} = 15.4 \text{ W} \frac{(273 + 100) \text{ K}}{(273 + 20) \text{ K}} = 19.6 \text{ W}$$

(a) $|Q_h| = W_{\text{eng}} + |Q_c|$

The useful power output is $\frac{W_{\text{eng}}}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{|Q_c|}{\Delta t} = 19.6 \text{ W} - 15.4 \text{ W} = \boxed{4.20 \text{ W}}$

(b) $|Q_h| = \left(\frac{|Q_h|}{\Delta t} \right) \Delta t = mL_V$ $m = \frac{|Q_h|}{\Delta t} \frac{\Delta t}{L_V} = (19.6 \text{ J/s}) \left(\frac{3600 \text{ s}}{2.26 \times 10^6 \text{ J/kg}} \right) = \boxed{3.12 \times 10^{-2} \text{ kg}}$

P22.16 The Carnot summer efficiency is
$$e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 + 20) \text{ K}}{(273 + 350) \text{ K}} = 0.530$$

And in winter,
$$e_{c,w} = 1 - \frac{283}{623} = 0.546$$

Then the actual winter efficiency is
$$0.320 \left(\frac{0.546}{0.530} \right) = \boxed{0.330} \text{ or } \boxed{33.0\%}$$

P22.17 (a) In an adiabatic process, $P_f V_f^\gamma = P_i V_i^\gamma$. Also,
$$\left(\frac{P_f V_f}{T_f} \right)^\gamma = \left(\frac{P_i V_i}{T_i} \right)^\gamma.$$

Dividing the second equation by the first yields
$$T_f = T_i \left(\frac{P_f}{P_i} \right)^{(\gamma-1)/\gamma}.$$

Since $\gamma = \frac{5}{3}$ for Argon, $\frac{\gamma-1}{\gamma} = \frac{2}{5} = 0.400$ and we have

$$T_f = (1\,073 \text{ K}) \left(\frac{300 \times 10^3 \text{ Pa}}{1.50 \times 10^6 \text{ Pa}} \right)^{0.400} = \boxed{564 \text{ K}}.$$

(b) $\Delta E_{\text{int}} = nC_V \Delta T = Q - W_{\text{eng}} = 0 - W_{\text{eng}}$, so $W_{\text{eng}} = -nC_V \Delta T$,

and the power output is

$$\begin{aligned} \mathcal{P} &= \frac{W_{\text{eng}}}{t} = \frac{-nC_V \Delta T}{t} \text{ or} \\ &= \frac{(-80.0 \text{ kg}) \left(\frac{1.00 \text{ mol}}{0.0399 \text{ kg}} \right) \left(\frac{3}{2} \right) (8.314 \text{ J/mol} \cdot \text{K}) (564 - 1\,073) \text{ K}}{60.0 \text{ s}} \\ \mathcal{P} &= 2.12 \times 10^5 \text{ W} = \boxed{212 \text{ kW}} \end{aligned}$$

(c)
$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{564 \text{ K}}{1\,073 \text{ K}} = 0.475 \text{ or } \boxed{47.5\%}$$

P22.18 (a)
$$e_{\text{max}} = 1 - \frac{T_c}{T_h} = 1 - \frac{278}{293} = 5.12 \times 10^{-2} = \boxed{5.12\%}$$

(b)
$$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = 75.0 \times 10^6 \text{ J/s}$$

Therefore,
$$W_{\text{eng}} = (75.0 \times 10^6 \text{ J/s})(3\,600 \text{ s/h}) = 2.70 \times 10^{11} \text{ J/h}$$

From $e = \frac{W_{\text{eng}}}{|Q_h|}$ we find
$$|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{2.70 \times 10^{11} \text{ J/h}}{5.12 \times 10^{-2}} = 5.27 \times 10^{12} \text{ J/h} = \boxed{5.27 \text{ TJ/h}}$$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

***P22.19** (a)
$$e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{h1}} = \frac{e_1 Q_{1h} + e_2 Q_{2h}}{Q_{h1}}$$

Now $Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng1}} = Q_{h1} - e_1 Q_{1h}$.

So
$$e = \frac{e_1 Q_{1h} + e_2 (Q_{1h} - e_1 Q_{1h})}{Q_{1h}} = \boxed{e_1 + e_2 - e_1 e_2}.$$

(b)
$$e = e_1 + e_2 - e_1 e_2 = 1 - \frac{T_i}{T_h} + 1 - \frac{T_c}{T_i} - \left(1 - \frac{T_i}{T_h}\right) \left(1 - \frac{T_c}{T_i}\right) = 2 - \frac{T_i}{T_h} - \frac{T_c}{T_i} - 1 + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}$$

The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency.

(c) With $W_{\text{eng2}} = W_{\text{eng1}}$,
$$e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{2W_{\text{eng1}}}{Q_{1h}} = 2e_1$$

$$1 - \frac{T_c}{T_h} = 2 \left(1 - \frac{T_i}{T_h}\right)$$

$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$

$$2T_i = T_h + T_c$$

$$\boxed{T_i = \frac{1}{2}(T_h + T_c)}$$

(d)
$$e_1 = e_2 = 1 - \frac{T_i}{T_h} = 1 - \frac{T_c}{T_i}$$

$$T_i^2 = T_c T_h$$

$$\boxed{T_i = (T_h T_c)^{1/2}}$$

P22.20 The work output is $W_{\text{eng}} = \frac{1}{2} m_{\text{train}} (5.00 \text{ m/s})^2$.

We are told
$$e = \frac{W_{\text{eng}}}{Q_h}$$

$$0.200 = \frac{1}{2} m_t \frac{(5.00 \text{ m/s})^2}{Q_h}$$

and
$$e_C = 1 - \frac{300 \text{ K}}{T_h} = \frac{1}{2} m_t \frac{(6.50 \text{ m/s})^2}{Q_h}.$$

Substitute $Q_h = \frac{1}{2} m_t \frac{(5.00 \text{ m/s})^2}{0.200}$.

Then,
$$1 - \frac{300 \text{ K}}{T_h} = 0.200 \left(\frac{\frac{1}{2} m_t (6.50 \text{ m/s})^2}{\frac{1}{2} m_t (5.00 \text{ m/s})^2} \right)$$

$$1 - \frac{300 \text{ K}}{T_h} = 0.338$$

$$T_h = \frac{300 \text{ K}}{0.662} = \boxed{453 \text{ K}}$$

P22.21 For the Carnot engine, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300 \text{ K}}{750 \text{ K}} = 0.600$.

Also,
$$e_c = \frac{W_{\text{eng}}}{|Q_h|}.$$

so
$$|Q_h| = \frac{W_{\text{eng}}}{e_c} = \frac{150 \text{ J}}{0.600} = 250 \text{ J}.$$

and
$$|Q_c| = |Q_h| - W_{\text{eng}} = 250 \text{ J} - 150 \text{ J} = 100 \text{ J}.$$

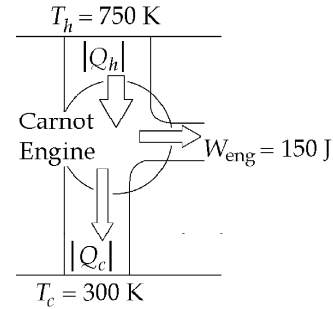


FIG. P22.21

(a)
$$|Q_h| = \frac{W_{\text{eng}}}{e_s} = \frac{150 \text{ J}}{0.700} = \boxed{214 \text{ J}}$$

$$|Q_c| = |Q_h| - W_{\text{eng}} = 214 \text{ J} - 150 \text{ J} = \boxed{64.3 \text{ J}}$$

(b)
$$|Q_{h,\text{net}}| = 214 \text{ J} - 250 \text{ J} = \boxed{-35.7 \text{ J}}$$

$$|Q_{c,\text{net}}| = 64.3 \text{ J} - 100 \text{ J} = \boxed{-35.7 \text{ J}}$$

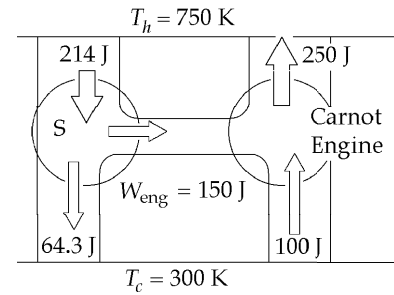


FIG. P22.21(b)

(c) For engine S: $|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_s} - W_{\text{eng}}.$

so
$$W_{\text{eng}} = \frac{|Q_c|}{\frac{1}{e_s} - 1} = \frac{100 \text{ J}}{\frac{1}{0.700} - 1} = \boxed{233 \text{ J}}.$$

and
$$|Q_h| = |Q_c| + W_{\text{eng}} = 233 \text{ J} + 100 \text{ J} = \boxed{333 \text{ J}}.$$

(d)
$$|Q_{h,\text{net}}| = 333 \text{ J} - 250 \text{ J} = \boxed{83.3 \text{ J}}$$

$$W_{\text{net}} = 233 \text{ J} - 150 \text{ J} = \boxed{83.3 \text{ J}}$$

$$|Q_{c,\text{net}}| = \boxed{0}$$

The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

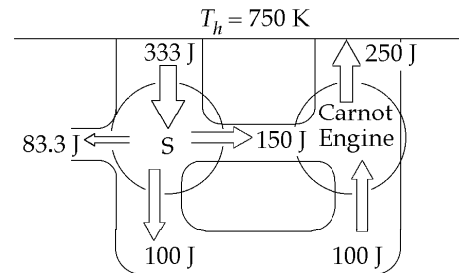


FIG. P22.21(d)

(e) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0.$

For the reservoirs,
$$\Delta S_h = -\frac{|Q_h|}{T_h} \text{ and } \Delta S_c = +\frac{|Q_c|}{T_c}.$$

Thus,
$$\Delta S_{\text{total}} = \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3 \text{ J}}{750 \text{ K}} + \frac{0}{300 \text{ K}} = \boxed{-0.111 \text{ J/K}}.$$

A decrease in total entropy is impossible.

P22.22 (a) First, consider the adiabatic process $D \rightarrow A$:

$$P_D V_D^\gamma = P_A V_A^\gamma \text{ so } P_D = P_A \left(\frac{V_A}{V_D} \right)^\gamma = 1\,400 \text{ kPa} \left(\frac{10.0 \text{ L}}{15.0 \text{ L}} \right)^{5/3} = \boxed{712 \text{ kPa}}.$$

$$\text{Also } \left(\frac{nRT_D}{V_D} \right) V_D^\gamma = \left(\frac{nRT_A}{V_A} \right) V_A^\gamma$$

$$\text{or } T_D = T_A \left(\frac{V_A}{V_D} \right)^{\gamma-1} = 720 \text{ K} \left(\frac{10.0}{15.0} \right)^{2/3} = \boxed{549 \text{ K}}.$$

Now, consider the isothermal process $C \rightarrow D$: $T_C = T_D = \boxed{549 \text{ K}}$.

$$P_C = P_D \left(\frac{V_D}{V_C} \right) = \left[P_A \left(\frac{V_A}{V_D} \right)^\gamma \right] \left(\frac{V_D}{V_C} \right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{1\,400 \text{ kPa} (10.0 \text{ L})^{5/3}}{24.0 \text{ L} (15.0 \text{ L})^{2/3}} = \boxed{445 \text{ kPa}}$$

Next, consider the adiabatic process $B \rightarrow C$: $P_B V_B^\gamma = P_C V_C^\gamma$.

$$\text{But, } P_C = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}} \text{ from above. Also considering the isothermal process, } P_B = P_A \left(\frac{V_A}{V_B} \right).$$

$$\text{Hence, } P_A \left(\frac{V_A}{V_B} \right) V_B^\gamma = \left(\frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}} \right) V_C^\gamma \text{ which reduces to } V_B = \frac{V_A V_C}{V_D} = \frac{10.0 \text{ L} (24.0 \text{ L})}{15.0 \text{ L}} = \boxed{16.0 \text{ L}}.$$

$$\text{Finally, } P_B = P_A \left(\frac{V_A}{V_B} \right) = 1\,400 \text{ kPa} \left(\frac{10.0 \text{ L}}{16.0 \text{ L}} \right) = \boxed{875 \text{ kPa}}.$$

State	$P(\text{kPa})$	$V(\text{L})$	$T(\text{K})$
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b) For the isothermal process $A \rightarrow B$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

$$\text{so } Q = -W = nRT \ln \left(\frac{V_B}{V_A} \right) = 2.34 \text{ mol} (8.314 \text{ J/mol} \cdot \text{K}) (720 \text{ K}) \ln \left(\frac{16.0}{10.0} \right) = \boxed{+6.58 \text{ kJ}}.$$

For the adiabatic process $B \rightarrow C$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V (T_C - T_B) = 2.34 \text{ mol} \left[\frac{3}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] (549 - 720) \text{ K} = \boxed{-4.98 \text{ kJ}}$$

$$\text{and } W = -Q + \Delta E_{\text{int}} = 0 + (-4.98 \text{ kJ}) = \boxed{-4.98 \text{ kJ}}.$$

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For the isothermal process $C \rightarrow D$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

and $Q = -W = nRT \ln\left(\frac{V_D}{V_C}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol} \cdot \text{K})(549 \text{ K}) \ln\left(\frac{15.0}{24.0}\right) = \boxed{-5.02 \text{ kJ}}$.

Finally, for the adiabatic process $D \rightarrow A$: $Q = \boxed{0}$

$\Delta E_{\text{int}} = nC_V(T_A - T_D) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol} \cdot \text{K})\right](720 - 549) \text{ K} = \boxed{+4.98 \text{ kJ}}$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98 \text{ kJ} = \boxed{+4.98 \text{ kJ}}$.

Process	$Q(\text{kJ})$	$W(\text{kJ})$	$\Delta E_{\text{int}}(\text{kJ})$
$A \rightarrow B$	+6.58	-6.58	0
$B \rightarrow C$	0	-4.98	-4.98
$C \rightarrow D$	-5.02	+5.02	0
$D \rightarrow A$	0	+4.98	+4.98
$ABCD A$	+1.56	-1.56	0

The work done *by* the engine is the negative of the work input. The output work W_{eng} is given by the work column in the table with all signs reversed.

(c) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A \rightarrow B}} = \frac{1.56 \text{ kJ}}{6.58 \text{ kJ}} = 0.237 \text{ or } \boxed{23.7\%}$

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237 \text{ or } \boxed{23.7\%}$

P22.23 $(\text{COP})_{\text{refrig}} = \frac{T_c}{\Delta T} = \frac{270}{30.0} = \boxed{9.00}$

P22.24 $(\text{COP})_{\text{heat pump}} = \frac{|Q_c| + W}{W} = \frac{T_h}{\Delta T} = \frac{295}{25} = \boxed{11.8}$

P22.25 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = |Q_h| - |Q_c| = |Q_c| \left[\frac{|Q_h|}{|Q_c|} - 1 \right]$.

We have already shown that for a Carnot cycle (and only for a Carnot cycle) $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$.

Therefore, $W = |Q_c| \left[\frac{T_h - T_c}{T_c} \right]$.

(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \frac{|Q_c|}{W}$.

Using the result from part (a), this becomes $\text{COP} = \frac{T_c}{T_h - T_c}$.

P22.26 $\text{COP} = 0.100 \text{COP}_{\text{Carnot cycle}}$

or $\frac{|Q_h|}{W} = 0.100 \left(\frac{|Q_h|}{W} \right)_{\text{Carnot cycle}} = 0.100 \left(\frac{1}{\text{Carnot efficiency}} \right)$

$$\frac{|Q_h|}{W} = 0.100 \left(\frac{T_h}{T_h - T_c} \right) = 0.100 \left(\frac{293 \text{ K}}{293 \text{ K} - 268 \text{ K}} \right) = 1.17$$

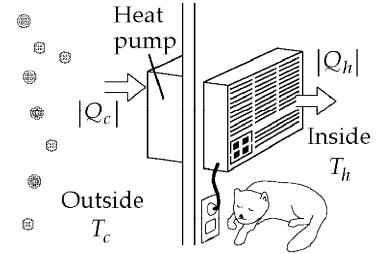


FIG. P22.26

Thus, 1.17 joules of energy enter the room by heat for each joule of work done.

P22.27 $(\text{COP})_{\text{Carnot refrig}} = \frac{T_c}{\Delta T} = \frac{4.00}{289} = 0.0138 = \frac{|Q_c|}{W}$
 $\therefore W = \boxed{72.2 \text{ J}}$ per 1 J energy removed by heat.

P22.28 A Carnot refrigerator runs on minimum power.

For it: $\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$ so $\frac{Q_h/t}{T_h} = \frac{Q_c/t}{T_c}$.

Solving part (b) first:

(b) $\frac{Q_h}{t} = \frac{Q_c}{t} \left(\frac{T_h}{T_c} \right) = (8.00 \text{ MJ/h}) \left(\frac{298 \text{ K}}{273 \text{ K}} \right) = (8.73 \times 10^6 \text{ J/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = \boxed{2.43 \text{ kW}}$

(a) $\frac{W}{t} = \frac{Q_h}{t} - \frac{Q_c}{t} = 2.43 \text{ kW} - \frac{8.00 \times 10^6 \text{ J/h}}{3600 \text{ s/h}} = \boxed{204 \text{ W}}$

P22.29 $e = \frac{W}{Q_h} = 0.350$ $W = 0.350 Q_h$
 $Q_h = W + Q_c$ $Q_c = 0.650 Q_h$
 $\text{COP}(\text{refrigerator}) = \frac{Q_c}{W} = \frac{0.650 Q_h}{0.350 Q_h} = \boxed{1.86}$

***P22.30** To have the same efficiencies as engines, $1 - \frac{T_{\text{cp}}}{T_{\text{hp}}} = 1 - \frac{T_{\text{cr}}}{T_{\text{hr}}}$ the pump and refrigerator must operate

between reservoirs with the same ratio $\frac{T_{\text{cp}}}{T_{\text{hp}}} = \frac{T_{\text{cr}}}{T_{\text{hr}}}$, which we define as r . Now $\text{COP}_p = 1.50 \text{COP}_r$

becomes $\frac{T_{\text{hp}}}{T_{\text{hp}} - T_{\text{cp}}} = \frac{3}{2} \frac{T_{\text{cr}}}{T_{\text{hr}} - T_{\text{cr}}}$ or $\frac{T_{\text{hp}}}{T_{\text{hp}} - r T_{\text{hp}}} = \frac{3}{2} \frac{r T_{\text{hr}}}{T_{\text{hr}} - r T_{\text{hr}}}$, $\frac{2}{1-r} = \frac{3r}{1-r}$, $r = \frac{2}{3}$.

(a) $\text{COP}_r = \frac{r}{1-r} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \boxed{2.00}$

(b) $\text{COP}_p = \frac{1}{1-r} = \frac{1}{1-\frac{2}{3}} = \boxed{3.00}$

(c) $e = 1 - r = 1 - \frac{2}{3} = \boxed{33.3\%}$

Section 22.5 Gasoline and Diesel Engines

P22.31 (a) $P_i V_i^\gamma = P_f V_f^\gamma$

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma = (3.00 \times 10^6 \text{ Pa}) \left(\frac{50.0 \text{ cm}^3}{300 \text{ cm}^3} \right)^{1.40} = \boxed{244 \text{ kPa}}$$

(b) $W = \int_{V_i}^{V_f} P dV$ $P = P_i \left(\frac{V_i}{V} \right)^\gamma$

Integrating,

$$W = \left(\frac{1}{\gamma - 1} \right) P_i V_i \left[1 - \left(\frac{V_i}{V_f} \right)^{\gamma-1} \right] = (2.50) (3.00 \times 10^6 \text{ Pa}) (5.00 \times 10^{-5} \text{ m}^3) \left[1 - \left(\frac{50.0 \text{ cm}^3}{300 \text{ cm}^3} \right)^{0.400} \right]$$

$$= \boxed{192 \text{ J}}$$

P22.32 Compression ratio = 6.00, $\gamma = 1.40$

(a) Efficiency of an Otto-engine $e = 1 - \left(\frac{V_2}{V_1} \right)^{\gamma-1}$

$$e = 1 - \left(\frac{1}{6.00} \right)^{0.400} = \boxed{51.2\%}$$

(b) If actual efficiency $e' = 15.0\%$ losses in system are $e - e' = \boxed{36.2\%}$.

P22.33 $e_{\text{Otto}} = 1 - \frac{1}{(V_1/V_2)^{\gamma-1}} = 1 - \frac{1}{(6.20)^{(7/5-1)}} = 1 - \frac{1}{(6.20)^{0.400}}$

$$e_{\text{Otto}} = 0.518$$

We have assumed the fuel-air mixture to behave like a diatomic gas.

Now $e = \frac{W_{\text{eng}}}{Q_h} = \frac{W_{\text{eng}}/t}{Q_h/t}$

$$\frac{Q_h}{t} = \frac{W_{\text{eng}}/t}{e} = 102 \text{ hp} \frac{746 \text{ W/1 hp}}{0.518}$$

$$\frac{Q_h}{t} = \boxed{146 \text{ kW}}$$

$$Q_h = W_{\text{eng}} + |Q_c|$$

$$\frac{|Q_c|}{t} = \frac{Q_h}{t} - \frac{W_{\text{eng}}}{t}$$

$$\frac{|Q_c|}{t} = 146 \times 10^3 \text{ W} - 102 \text{ hp} \left(\frac{746 \text{ W}}{1 \text{ hp}} \right) = \boxed{70.8 \text{ kW}}$$

P22.34 (a), (b) The quantity of gas is

$$n = \frac{P_A V_A}{RT_A} = \frac{(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.0205 \text{ mol}$$

$$E_{\text{int}, A} = \frac{5}{2} nRT_A = \frac{5}{2} P_A V_A = \frac{5}{2} (100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3) = \boxed{125 \text{ J}}$$

$$\text{In process AB, } P_B = P_A \left(\frac{V_A}{V_B} \right)^\gamma = (100 \times 10^3 \text{ Pa})(8.00)^{1.40} = \boxed{1.84 \times 10^6 \text{ Pa}}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{(1.84 \times 10^6 \text{ Pa})(500 \times 10^{-6} \text{ m}^3 / 8.00)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = \boxed{673 \text{ K}}$$

$$E_{\text{int}, B} = \frac{5}{2} nRT_B = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(673 \text{ K}) = \boxed{287 \text{ J}}$$

$$\text{so } \Delta E_{\text{int}, AB} = 287 \text{ J} - 125 \text{ J} = \boxed{162 \text{ J}} = Q - W_{\text{out}} = 0 - W_{\text{out}} \quad W_{AB} = \boxed{-162 \text{ J}}$$

Process BC takes us to:

$$P_C = \frac{nRT_C}{V_C} = \frac{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K})}{62.5 \times 10^{-6} \text{ m}^3} = \boxed{2.79 \times 10^6 \text{ Pa}}$$

$$E_{\text{int}, C} = \frac{5}{2} nRT_C = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K}) = \boxed{436 \text{ J}}$$

$$E_{\text{int}, BC} = 436 \text{ J} - 287 \text{ J} = \boxed{149 \text{ J}} = Q - W_{\text{out}} = Q - 0$$

$$Q_{BC} = \boxed{149 \text{ J}}$$

In process CD:

$$P_D = P_C \left(\frac{V_C}{V_D} \right)^\gamma = (2.79 \times 10^6 \text{ Pa}) \left(\frac{1}{8.00} \right)^{1.40} = \boxed{1.52 \times 10^5 \text{ Pa}}$$

$$T_D = \frac{P_D V_D}{nR} = \frac{(1.52 \times 10^5 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = \boxed{445 \text{ K}}$$

$$E_{\text{int}, D} = \frac{5}{2} nRT_D = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(445 \text{ K}) = \boxed{190 \text{ J}}$$

$$\Delta E_{\text{int}, CD} = 190 \text{ J} - 436 \text{ J} = \boxed{-246 \text{ J}} = Q - W_{\text{out}} = 0 - W_{\text{out}}$$

$$W_{CD} = \boxed{246 \text{ J}}$$

$$\text{and } \Delta E_{\text{int}, DA} = E_{\text{int}, A} - E_{\text{int}, D} = 125 \text{ J} - 190 \text{ J} = \boxed{-65.0 \text{ J}} = Q - W_{\text{out}} = Q - 0$$

$$Q_{DA} = \boxed{-65.0 \text{ J}}$$

continued on next page

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For the entire cycle, $\Delta E_{\text{int, net}} = 162 \text{ J} + 149 - 246 - 65.0 = \boxed{0}$. The net work is

$$W_{\text{eng}} = -162 \text{ J} + 0 + 246 \text{ J} + 0 = \boxed{84.3 \text{ J}}$$

$$Q_{\text{net}} = 0 + 149 \text{ J} + 0 - 65.0 \text{ J} = \boxed{84.3 \text{ J}}$$

The tables look like:

State	$T(\text{K})$	$P(\text{kPa})$	$V(\text{cm}^3)$	$E_{\text{int}} (\text{J})$
<i>A</i>	293	100	500	125
<i>B</i>	673	1 840	62.5	287
<i>C</i>	1 023	2 790	62.5	436
<i>D</i>	445	152	500	190
<i>A</i>	293	100	500	125

Process	$Q(\text{J})$	output $W(\text{J})$	$\Delta E_{\text{int}} (\text{J})$
<i>AB</i>	0	-162	162
<i>BC</i>	149	0	149
<i>CD</i>	0	246	-246
<i>DA</i>	-65.0	0	-65.0
<i>ABCD</i>	84.3	84.3	0

(c) The input energy is $Q_h = \boxed{149 \text{ J}}$, the waste is $|Q_c| = \boxed{65.0 \text{ J}}$, and $W_{\text{eng}} = \boxed{84.3 \text{ J}}$.

(d) The efficiency is: $e = \frac{W_{\text{eng}}}{Q_h} = \frac{84.3 \text{ J}}{149 \text{ J}} = \boxed{0.565}$.

(e) Let f represent the angular speed of the crankshaft. Then $\frac{f}{2}$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1\,000 \text{ J/s} = \left(\frac{f}{2}\right)(84.3 \text{ J/cycle})$$

$$f = \frac{2\,000 \text{ J/s}}{84.3 \text{ J/cycle}} = 23.7 \text{ rev/s} = \boxed{1.42 \times 10^3 \text{ rev/min}}$$

Section 22.6 Entropy

P22.35 For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500 \text{ kg})(3.33 \times 10^5 \text{ J/kg})}{273 \text{ K}} = \boxed{-610 \text{ J/K}}.$$

P22.36 At a constant temperature of 4.20 K,

$$\Delta S = \frac{\Delta Q}{T} = \frac{L_v}{4.20 \text{ K}} = \frac{20.5 \text{ kJ/kg}}{4.20 \text{ K}}$$

$$\Delta S = \boxed{4.88 \text{ kJ/kg} \cdot \text{K}}$$

P22.37
$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc dT}{T} = mc \ln \left(\frac{T_f}{T_i} \right)$$

$$\Delta S = 250 \text{ g} (1.00 \text{ cal/g} \cdot ^\circ\text{C}) \ln \left(\frac{353}{293} \right) = 46.6 \text{ cal/K} = \boxed{195 \text{ J/K}}$$

- *P22.38** (a) The process is isobaric because it takes place under constant atmospheric pressure. As described by Newton's third law, the stewing syrup must exert the same force on the air as the air exerts on it. The heating process is not adiabatic (energy goes in by heat), isothermal (T goes up), isovolumetric (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. The process would then also be eternal, and impractical for food production.
- (b) The final temperature is

$$220^\circ\text{F} = 212^\circ\text{F} + 8^\circ\text{F} = 100^\circ\text{C} + 8^\circ\text{F} \left(\frac{100 - 0^\circ\text{C}}{212 - 32^\circ\text{F}} \right) = 104^\circ\text{C}.$$

For the mixture,

$$Q = m_1 c_1 \Delta T + m_2 c_2 \Delta T = (900 \text{ g } 1 \text{ cal/g} \cdot ^\circ\text{C} + 930 \text{ g } 0.299 \text{ cal/g} \cdot ^\circ\text{C}) (104.4^\circ\text{C} - 23^\circ\text{C})$$

$$= 9.59 \times 10^4 \text{ cal} = \boxed{4.02 \times 10^5 \text{ J}}$$

- (c) Consider the reversible heating process described in part (a):

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{(m_1 c_1 + m_2 c_2) dT}{T} = (m_1 c_1 + m_2 c_2) \ln \frac{T_f}{T_i}$$

$$= [900(1) + 930(0.299)] (\text{cal/g} \cdot ^\circ\text{C}) \left(\frac{4.186 \text{ J}}{1 \text{ cal}} \right) \left(\frac{1^\circ\text{C}}{1 \text{ K}} \right) \ln \left(\frac{273 + 104}{273 + 23} \right)$$

$$= (4930 \text{ J/K}) 0.243 = \boxed{1.20 \times 10^3 \text{ J/K}}$$

- *P22.39** We take data from the description of Figure 20.2 in section 20.3, and we assume a constant specific heat for each phase. As the ice is warmed from -12°C to 0°C , its entropy increases by

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\text{ K}}^{273\text{ K}} \frac{mc_{\text{ice}}dT}{T} = mc_{\text{ice}} \int_{261\text{ K}}^{273\text{ K}} T^{-1}dT = mc_{\text{ice}} \ln T \Big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S = 0.0270\text{ kg}(2090\text{ J/kg}\cdot^{\circ}\text{C})(\ln 273\text{ K} - \ln 261\text{ K}) = 0.0270\text{ kg}(2090\text{ J/kg}\cdot^{\circ}\text{C})\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.54\text{ J/K}$$

As the ice melts its entropy change is

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{0.0270\text{ kg}(3.33 \times 10^5\text{ J/kg})}{273\text{ K}} = 32.9\text{ J/K}$$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{mc_{\text{liquid}}dT}{T} = mc_{\text{liquid}} \ln\left(\frac{T_f}{T_i}\right) = 0.0270\text{ kg}(4186\text{ J/kg}\cdot^{\circ}\text{C})\ln\left(\frac{373}{273}\right) = 35.3\text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + mc_{\text{steam}} \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.0270\text{ kg}(2.26 \times 10^6\text{ J/kg})}{373\text{ K}} + 0.0270\text{ kg}(2010\text{ J/kg}\cdot^{\circ}\text{C})\ln\left(\frac{388}{373}\right) = 164\text{ J/K} + 2.14\text{ J/K}$$

The total entropy change is

$$(2.54 + 32.9 + 35.3 + 164 + 2.14)\text{ J/K} = \boxed{236\text{ J/K}}.$$

We could equally well have taken the values for specific heats and latent heats from Tables 20.1 and 20.2. For steam at constant pressure, the molar specific heat in Table 21.2 implies a specific heat of

$$(35.4\text{ J/mol}\cdot\text{K})\left(\frac{1\text{ mol}}{0.018\text{ kg}}\right) = 1970\text{ J/kg}\cdot\text{K}, \text{ nearly agreeing with } 2010\text{ J/kg}\cdot\text{K}.$$

Section 22.7 Entropy Changes in Irreversible Processes

P22.40 $\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1000}{290} - \frac{1000}{5700}\right)\text{ J/K} = \boxed{3.27\text{ J/K}}$

- P22.41** The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293}\text{ J/K} = \boxed{1.02\text{ kJ/K}}.$$

P22.42 $c_{\text{iron}} = 448 \text{ J/kg} \cdot ^\circ\text{C}$; $c_{\text{water}} = 4186 \text{ J/kg} \cdot ^\circ\text{C}$

$$Q_{\text{cold}} = -Q_{\text{hot}}: 4.00 \text{ kg}(4186 \text{ J/kg} \cdot ^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00 \text{ kg})(448 \text{ J/kg} \cdot ^\circ\text{C})(T_f - 900^\circ\text{C})$$

which yields $T_f = 33.2^\circ\text{C} = 306.2 \text{ K}$

$$\Delta S = \int_{283 \text{ K}}^{306.2 \text{ K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1173 \text{ K}}^{306.2 \text{ K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1173}\right)$$

$$\Delta S = (4186 \text{ J/kg} \cdot \text{K})(4.00 \text{ kg})(0.0788) + (448 \text{ J/kg} \cdot \text{K})(1.00 \text{ kg})(-1.34)$$

$$\Delta S = \boxed{718 \text{ J/K}}$$

P22.43 Sitting here writing, I convert chemical energy, in ordered molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2500 \text{ kcal/d} = \frac{2500 \times 10^3 \text{ cal}}{86400 \text{ s}} \left(\frac{4186 \text{ J}}{1 \text{ cal}} \right) = 120 \text{ W}.$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120 \text{ W}}{293 \text{ K}} = 0.4 \text{ W/K} \sim \boxed{1 \text{ W/K}}.$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P22.44 (a) $V = \frac{nRT_i}{P_i} = \frac{(40.0 \text{ g})(8.314 \text{ J/mol} \cdot \text{K})(473 \text{ K})}{(39.9 \text{ g/mol})(100 \times 10^3 \text{ Pa})} = 39.4 \times 10^{-3} \text{ m}^3 = \boxed{39.4 \text{ L}}$

(b) $\Delta E_{\text{int}} = nC_V \Delta T = \left(\frac{40.0 \text{ gm}}{39.9 \text{ g/mol}} \right) \left[\frac{3}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] (-200^\circ\text{C}) = \boxed{-2.50 \text{ kJ}}$

(c) $W = 0$ so $Q = \Delta E_{\text{int}} = \boxed{-2.50 \text{ kJ}}$

(d) $\Delta S_{\text{argon}} = \int_i^f \frac{dQ}{T} = nC_V \ln\left(\frac{T_f}{T_i}\right)$
 $= \left(\frac{40.0 \text{ g}}{39.9 \text{ g/mol}} \right) \left[\frac{3}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] \ln\left(\frac{273}{473}\right) = \boxed{-6.87 \text{ J/K}}$

(e) $\Delta S_{\text{bath}} = \frac{2.50 \text{ kJ}}{273 \text{ K}} = \boxed{+9.16 \text{ J/K}}$

The total change in entropy is

$$\Delta S_{\text{total}} = \Delta S_{\text{argon}} + \Delta S_{\text{bath}} = -6.87 \text{ J/K} + 9.16 \text{ J/K} = +2.29 \text{ J/K}$$

$\Delta S_{\text{total}} > 0$ for this irreversible process.

P22.45 $\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) = R \ln 2 = \boxed{5.76 \text{ J/K}}$

There is no change in temperature.

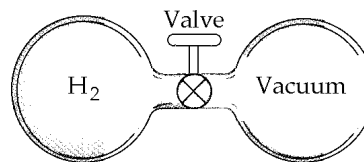


FIG. P22.45

P22.46 $\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) = (0.0440)(2)R \ln 2$

$\Delta S = 0.0880(8.314) \ln 2 = \boxed{0.507 \text{ J/K}}$

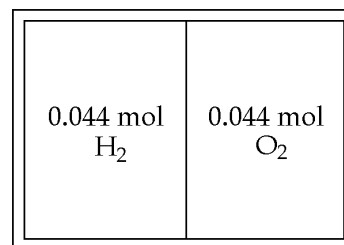


FIG. P22.46

P22.47 For any infinitesimal step in a process on an ideal gas,

$$dE_{\text{int}} = dQ + dW : \quad dQ = dE_{\text{int}} - dW = nC_V dT + PdV = nC_V dT + \frac{nRTdV}{V}$$

and
$$\frac{dQ}{T} = nC_V \frac{dT}{T} + nR \frac{dV}{V}$$

If the whole process is reversible,
$$\Delta S = \int_i^f \frac{dQ_r}{T} = \int_i^f \left(nC_V \frac{dT}{T} + nR \frac{dV}{V} \right) = nC_V \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$$

Also, from the ideal gas law,
$$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i}$$

$$\begin{aligned} \Delta S &= (1.00 \text{ mol}) \left[\frac{3}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] \ln \left(\frac{(2.00)(0.0400)}{(1.00)(0.0250)} \right) + (1.00 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) \ln \left(\frac{0.0400}{0.0250} \right) \\ &= \boxed{18.4 \text{ J/K}} \end{aligned}$$

P22.48 $\Delta S = nC_V \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$

$$= (1.00 \text{ mol}) \left[\frac{5}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] \ln \left(\frac{2P \cdot 2V}{PV} \right) + (1.00 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) \ln \left(\frac{2V}{V} \right)$$

$\Delta S = \boxed{34.6 \text{ J/K}}$



Section 22.8 Entropy on a Microscopic Scale

P22.49 (a) A 12 can only be obtained one way 6 + 6

(b) A 7 can be obtained six ways: 6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6

P22.50 (a) The table is shown below. On the basis of the table, the most probable result of a toss is 2 heads and 2 tails.

(b) The most ordered state is the least likely state. Thus, on the basis of the table this is either all heads or all tails.

(c) The most disordered is the most likely state. Thus, this is 2 heads and 2 tails.

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

P22.51 (a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GRRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

Additional Problems

P22.52 The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount mgY , to the water from a stove at a temperature infinitesimally above 20.0°C . Then,

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{mgY}{T} = \frac{5\,000\text{ m}^3 (1\,000\text{ kg/m}^3) (9.80\text{ m/s}^2) (50.0\text{ m})}{293\text{ K}} = \boxed{8.36 \times 10^6\text{ J/K}}.$$

652 Heat Engines, Entropy, and the Second Law of Thermodynamics

- P22.53** (a) $\mathcal{P}_{\text{electric}} = \frac{H_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $H_{ET} = |Q|$.

Therefore,
$$\mathcal{P}_{\text{electric}} = \frac{Q}{\Delta t} = \boxed{5\,000\text{ W}}$$

- (b) For a heat pump,
$$(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295\text{ K}}{27\text{ K}} = 10.92$$

$$\text{Actual COP} = 0.6(10.92) = 6.55 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$\mathcal{P}_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\,000\text{ W}}{6.56} = \boxed{763\text{ W}}$$

- P22.54** $|Q_c| = mc\Delta T + mL + mc\Delta T =$

$$|Q_c| = 0.500\text{ kg}(4\,186\text{ J/kg}\cdot^\circ\text{C})(10^\circ\text{C}) + 0.500\text{ kg}(3.33 \times 10^5\text{ J/kg}) + 0.500\text{ kg}(2\,090\text{ J/kg}\cdot^\circ\text{C})(20^\circ\text{C})$$

$$|Q_c| = 2.08 \times 10^5\text{ J}$$

$$\frac{|Q_c|}{W} = \text{COP}_c(\text{refrigerator}) = \frac{T_c}{T_h - T_c}$$

$$W = \frac{|Q_c|(T_h - T_c)}{T_c} = \frac{(2.08 \times 10^5\text{ J})[20.0^\circ\text{C} - (-20.0^\circ\text{C})]}{(273 - 20.0)\text{ K}} = \boxed{32.9\text{ kJ}}$$

- P22.55** $\Delta S_{\text{hot}} = \frac{-1\,000\text{ J}}{600\text{ K}}$

$$\Delta S_{\text{cold}} = \frac{+750\text{ J}}{350\text{ K}}$$

- (a) $\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \boxed{0.476\text{ J/K}}$

- (b) $e_c = 1 - \frac{T_1}{T_2} = 0.417$

$$W_{\text{eng}} = e_c |Q_h| = 0.417(1\,000\text{ J}) = \boxed{417\text{ J}}$$

- (c) $W_{\text{net}} = 417\text{ J} - 250\text{ J} = 167\text{ J}$

$$T_1 \Delta S_U = 350\text{ K}(0.476\text{ J/K}) = \boxed{167\text{ J}}$$

- *P22.56** (a) The energy put into the engine by the hot reservoir is $dQ_h = mcdT_h$. The energy put into the cold reservoir by the engine is $|dQ_c| = -mcdT_c = (1-e)dQ_h = \left[1 - \left(1 - \frac{T_c}{T_h}\right)\right]mcdT_h$. Then

$$\begin{aligned} -\frac{dT_c}{T_c} &= \frac{dT_h}{T_h} \\ \int_{T_c}^{T_f} -\frac{dT}{T} &= \int_{T_h}^{T_f} \frac{dT}{T} \\ -\ln T \Big|_{T_c}^{T_f} &= \ln T \Big|_{T_h}^{T_f} \\ \ln \frac{T_c}{T_f} &= \ln \frac{T_f}{T_h} \\ T_f^2 &= T_c T_h \\ T_f &= (T_h T_c)^{1/2} \end{aligned}$$

- (b) The hot reservoir loses energy $|Q_h| = mc(T_h - T_f)$. The cold reservoir gains $|Q_c| = mc(T_f - T_c)$. Then $|Q_h| = W_{\text{eng}} + |Q_c|$.

$$\begin{aligned} W_{\text{eng}} &= mc(T_h - T_f) - mc(T_f - T_c) \\ &= mc(T_h - \sqrt{T_h T_c} - \sqrt{T_h T_c} + T_c) \\ &= mc(T_h - 2\sqrt{T_h T_c} + T_c) = mc(\sqrt{T_h} - \sqrt{T_c})^2 \end{aligned}$$

- P22.57** (a) For an isothermal process, $Q = nRT \ln\left(\frac{V_2}{V_1}\right)$
 Therefore, $Q_1 = nR(3T_i) \ln 2$
 and $Q_3 = nR(T_i) \ln\left(\frac{1}{2}\right)$
 For the constant volume processes, $Q_2 = \Delta E_{\text{int}, 2} = \frac{3}{2}nR(T_i - 3T_i)$
 and $Q_4 = \Delta E_{\text{int}, 4} = \frac{3}{2}nR(3T_i - T_i)$
 The net energy by heat transferred is then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

or

$$Q = \boxed{2nRT_i \ln 2}.$$

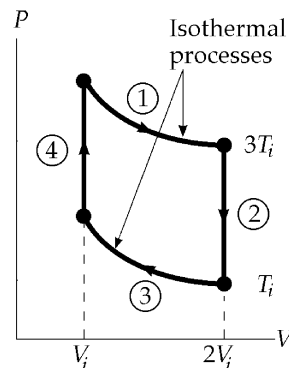


FIG. P22.57

- (b) A positive value for heat represents energy transferred into the system.
 Therefore, $|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$
 Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \text{ and } W_{\text{eng}} = Q$$

Therefore, the efficiency is

$$e_c = \frac{W_{\text{eng}}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2 \ln 2}{3(1 + \ln 2)} = \boxed{0.273}$$

$$\text{P22.58} \quad (a) \quad \frac{W_{\text{eng}}}{t} = 1.50 \times 10^8 \text{ W}_{(\text{electrical})}, Q = mL = \left[\frac{W_{\text{eng}}}{0.150} \right] \Delta t,$$

$$\text{and } L = 33.0 \text{ kJ/g} = 33.0 \times 10^6 \text{ J/kg}$$

$$m = \left[\frac{W_{\text{eng}}/t}{0.150} \right] \frac{\Delta t}{L}$$

$$m = \frac{(1.50 \times 10^8 \text{ W})(86400 \text{ s/day})}{0.150(33.0 \times 10^6 \text{ J/kg})(10^3 \text{ kg/metric ton})} = \boxed{2620 \text{ metric tons/day}}$$

$$(b) \quad \text{Cost} = (\$8.00/\text{metric ton})(2618 \text{ metric tons/day})(365 \text{ days/yr})$$

$$\text{Cost} = \boxed{\$7.65 \text{ million/year}}$$

- (c) First find the rate at which heat energy is discharged into the water. If the plant is 15.0% efficient in producing electrical energy then the rate of heat production is

$$\frac{|Q_c|}{t} = \left(\frac{W_{\text{eng}}}{t} \right) \left(\frac{1}{e} - 1 \right) = (1.50 \times 10^8 \text{ W}) \left(\frac{1}{0.150} - 1 \right) = 8.50 \times 10^8 \text{ W}.$$

$$\text{Then, } \frac{|Q_c|}{t} = \frac{mc\Delta T}{t} \text{ and}$$

$$\frac{m}{t} = \frac{\frac{|Q_c|}{t}}{c\Delta T} = \frac{8.50 \times 10^8 \text{ J/s}}{(4186 \text{ J/kg}\cdot^\circ\text{C})(5.00^\circ\text{C})} = \boxed{4.06 \times 10^4 \text{ kg/s}}.$$

$$\text{P22.59} \quad e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}}: \quad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{(1 - T_c/T_h)} = \frac{\mathcal{P}T_h}{T_h - T_c}$$

$$|Q_h| = W_{\text{eng}} + |Q_c|: \quad \frac{|Q_c|}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{W_{\text{eng}}}{\Delta t}$$

$$\frac{|Q_c|}{\Delta t} = \frac{\mathcal{P}T_h}{T_h - T_c} - \mathcal{P} = \frac{\mathcal{P}T_c}{T_h - T_c}$$

$$|Q_c| = mc\Delta T: \quad \frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t} \right) c\Delta T = \frac{\mathcal{P}T_c}{T_h - T_c}$$

$$\frac{\Delta m}{\Delta t} = \frac{\mathcal{P}T_c}{(T_h - T_c)c\Delta T}$$

$$\frac{\Delta m}{\Delta t} = \frac{(1.00 \times 10^9 \text{ W})(300 \text{ K})}{200 \text{ K}(4186 \text{ J/kg}\cdot^\circ\text{C})(6.00^\circ\text{C})} = \boxed{5.97 \times 10^4 \text{ kg/s}}$$

$$\text{P22.60} \quad e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}} \quad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\mathcal{P} T_h}{T_h - T_c}$$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{|Q_h|}{\Delta t}\right) - \mathcal{P} = \frac{\mathcal{P} T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$, where c is the specific heat of water.

Therefore,
$$\frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)c\Delta T = \frac{\mathcal{P} T_c}{T_h - T_c}$$

and
$$\frac{\Delta m}{\Delta t} = \boxed{\frac{\mathcal{P} T_c}{(T_h - T_c)c\Delta T}}$$

P22.61 (a) $35.0^\circ\text{F} = \frac{5}{9}(35.0 - 32.0)^\circ\text{C} = (1.67 + 273.15)\text{K} = 274.82\text{K}$

$98.6^\circ\text{F} = \frac{5}{9}(98.6 - 32.0)^\circ\text{C} = (37.0 + 273.15)\text{K} = 310.15\text{K}$

$$\Delta S_{\text{ice water}} = \int \frac{dQ}{T} = (453.6\text{ g})(1.00\text{ cal/g}\cdot\text{K}) \times \int_{274.82}^{310.15} \frac{dT}{T} = 453.6 \ln\left(\frac{310.15}{274.82}\right) = 54.86\text{ cal/K}$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00) \frac{(310.15 - 274.82)}{310.15} = -51.67\text{ cal/K}$$

$$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19\text{ cal/K}}$$

(b) $(453.6)(1)(T_F - 274.82) = (70.0 \times 10^3)(1)(310.15 - T_F)$

Thus,

$$(70.0 + 0.4536) \times 10^3 T_F = [(70.0)(310.15) + (0.4536)(274.82)] \times 10^3$$

and $T_F = 309.92\text{K} = 36.77^\circ\text{C} = \boxed{98.19^\circ\text{F}}$

$$\Delta S'_{\text{ice water}} = 453.6 \ln\left(\frac{309.92}{274.82}\right) = 54.52\text{ cal/K}$$

$$\Delta S'_{\text{body}} = -(70.0 \times 10^3) \ln\left(\frac{310.15}{309.92}\right) = -51.93\text{ cal/K}$$

$$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59\text{ cal/K}} \text{ which is less than the estimate in part (a).}$$

P22.62 (a) For the isothermal process AB , the work on the gas is

$$W_{AB} = -P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = -5(1.013 \times 10^5 \text{ Pa})(10.0 \times 10^{-3} \text{ m}^3) \ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = -8.15 \times 10^3 \text{ J}$$

where we have used $1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

and $1.00 \text{ L} = 1.00 \times 10^{-3} \text{ m}^3$

$$W_{BC} = -P_B \Delta V = -(1.013 \times 10^5 \text{ Pa})[(10.0 - 50.0) \times 10^{-3}] \text{ m}^3 = +4.05 \times 10^3 \text{ J}$$

$$W_{CA} = 0 \text{ and } W_{\text{eng}} = -W_{AB} - W_{BC} = 4.11 \times 10^3 \text{ J} = \boxed{4.11 \text{ kJ}}$$

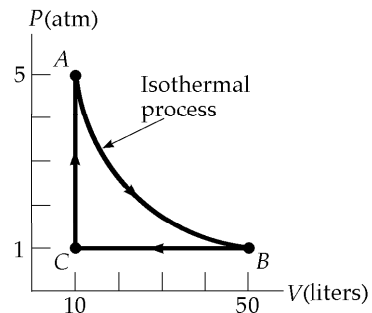


FIG. P22.62

(b) Since AB is an isothermal process, $\Delta E_{\text{int}, AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15 \times 10^3 \text{ J}$

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{(1.013 \times 10^5)(50.0 \times 10^{-3})}{R} = \frac{5.05 \times 10^3}{R}$$

$$\text{Also, } T_C = \frac{P_C V_C}{nR} = \frac{(1.013 \times 10^5)(10.0 \times 10^{-3})}{R} = \frac{1.01 \times 10^3}{R}$$

$$Q_{CA} = nC_V \Delta T = 1.00 \left(\frac{3}{2} R \right) \left(\frac{5.05 \times 10^3 - 1.01 \times 10^3}{R} \right) = 6.08 \text{ kJ}$$

so the total energy absorbed by heat is $Q_{AB} + Q_{CA} = 8.15 \text{ kJ} + 6.08 \text{ kJ} = \boxed{14.2 \text{ kJ}}$.

$$(c) \quad Q_{BC} = nC_P \Delta T = \frac{5}{2}(nR \Delta T) = \frac{5}{2}P_B \Delta V_{BC}$$

$$Q_{BC} = \frac{5}{2}(1.013 \times 10^5) [(10.0 - 50.0) \times 10^{-3}] = -1.01 \times 10^4 \text{ J} = \boxed{-10.1 \text{ kJ}}$$

$$(d) \quad e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB} + Q_{CA}} = \frac{4.11 \times 10^3 \text{ J}}{1.42 \times 10^4 \text{ J}} = 0.289 \text{ or } \boxed{28.9\%}$$

- *P22.63** Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is

$$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280 \text{ K}}{20 \text{ K}} = 14.0$$

- (a) Its actual COP is

$$0.400(14.0) = 5.60 = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{|Q_c/\Delta t|}{|Q_h/\Delta t| - |Q_c/\Delta t|}$$

$$5.60 \left| \frac{Q_h}{\Delta t} \right| - 5.60 \left| \frac{Q_c}{\Delta t} \right| = \left| \frac{Q_c}{\Delta t} \right|$$

$$5.60(10.0 \text{ kW}) = 6.60 \left| \frac{Q_c}{\Delta t} \right| \text{ and } \left| \frac{Q_c}{\Delta t} \right| = \boxed{8.48 \text{ kW}}$$

- (b) $|Q_h| = W_{\text{eng}} + |Q_c|$:

$$\frac{W_{\text{eng}}}{\Delta t} = \left| \frac{Q_h}{\Delta t} \right| - \left| \frac{Q_c}{\Delta t} \right| = 10.0 \text{ kW} - 8.48 \text{ kW} = \boxed{1.52 \text{ kW}}$$

- (c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h} = \frac{(10.0 \times 10^3 \text{ J/s})(3600 \text{ s})}{300 \text{ K}} = 1.20 \times 10^5 \text{ J/K}$$

The cold room decreases in entropy by

$$\Delta S = -\frac{|Q_c|}{T_c} = -\frac{(8.48 \times 10^3 \text{ J/s})(3600 \text{ s})}{280 \text{ K}} = -1.09 \times 10^5 \text{ J/K}$$

The net entropy change is positive, as it must be:

$$+1.20 \times 10^5 \text{ J/K} - 1.09 \times 10^5 \text{ J/K} = \boxed{1.09 \times 10^4 \text{ J/K}}$$

- (d) The new ideal COP is

$$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280 \text{ K}}{25 \text{ K}} = 11.2$$

We suppose the actual COP is

$$0.400(11.2) = 4.48$$

As a fraction of the original 5.60, this is $\frac{4.48}{5.60} = 0.800$, so the fractional change is to

$$\boxed{\text{drop by } 20.0\%}.$$

P22.64

(a)
$$W = \int_{V_i}^{V_f} P dV = nRT \int_{V_i}^{2V_i} \frac{dV}{V} = (1.00)RT \ln\left(\frac{2V_i}{V_i}\right) = \boxed{RT \ln 2}$$

- (b) $\boxed{\text{The second law refers to cycles.}}$

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P22.65 At point A, $P_i V_i = nRT_i$ and $n = 1.00 \text{ mol}$

At point B, $3P_i V_i = nRT_B$ so $T_B = 3T_i$

At point C, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point D, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and

$$C_P = \frac{5R}{2}:$$

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$

$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$

$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$

$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

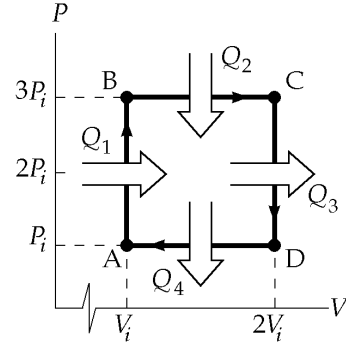


FIG. P22.65

(a) Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

(d) Carnot efficiency, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{T_i}{6T_i} = \boxed{0.833}$

***P22.66** $\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1} dT = nC_P \ln T \Big|_{T_i}^{T_f} = nC_P (\ln T_f - \ln T_i) = nC_P \ln \left(\frac{T_f}{T_i} \right)$

$$\Delta S = nC_P \ln \left(\frac{P V_f}{nR} \frac{nR}{P V_i} \right) = \boxed{nC_P \ln 3}$$

***P22.67** (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i, T_i represent the state of the gas before the isothermal expansion. Let P_C, V_C, T_i represent the state after this process, so that $P_i V_i = P_C V_C$. Let $P_i, 3V_i, T_f$ represent the state after the adiabatic compression.

Then $P_C V_C^\gamma = P_i (3V_i)^\gamma$

Substituting $P_C = \frac{P_i V_i}{V_C}$

gives $P_i V_i V_C^{\gamma-1} = P_i (3^\gamma V_i^\gamma)$

Then $V_C^{\gamma-1} = 3^\gamma V_i^{\gamma-1}$ and $\frac{V_C}{V_i} = 3^{\gamma/(\gamma-1)}$

continued on next page

The work output in the isothermal expansion is

$$W = \int_i^C P dV = nRT_i \int_i^C V^{-1} dV = nRT_i \ln\left(\frac{V_C}{V_i}\right) = nRT_i \ln(3^{\gamma/(\gamma-1)}) = nRT_i \left(\frac{\gamma}{\gamma-1}\right) \ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S = \frac{Q}{T} = nR \left(\frac{\gamma}{\gamma-1}\right) \ln 3$$

Since

$$C_p = \gamma C_V = C_V + R$$

we have

$$(\gamma-1)C_V = R, \quad C_V = \frac{R}{\gamma-1}$$

and

$$C_p = \frac{\gamma R}{\gamma-1}$$

Then the result is

$$\boxed{\Delta S = nC_p \ln 3}$$

- (b) The pair of processes considered here carry the gas from the initial state in Problem 66 to the final state there. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in Problem 66 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to Problems 66 and 67 (a) must be the same.

P22.68 Simply evaluate the maximum (Carnot) efficiency.

$$e_C = \frac{\Delta T}{T_h} = \frac{4.00 \text{ K}}{277 \text{ K}} = \boxed{0.0144}$$

The proposal does not merit serious consideration.

P22.69 The heat transfer over the paths CD and BA is zero since they are adiabatic.

Over path BC : $Q_{BC} = nC_p(T_C - T_B) > 0$

Over path DA : $Q_{DA} = nC_V(T_A - T_D) < 0$

Therefore, $|Q_c| = |Q_{DA}|$ and $Q_h = Q_{BC}$

The efficiency is then

$$e = 1 - \frac{|Q_c|}{Q_h} = 1 - \frac{(T_D - T_A)C_V}{(T_C - T_B)C_p}$$

$$e = 1 - \frac{1}{\gamma} \left[\frac{T_D - T_A}{T_C - T_B} \right]$$

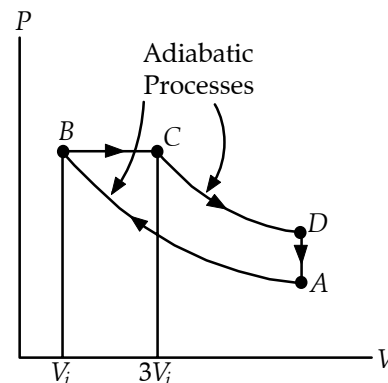


FIG. P22.69

P22.70 (a) Use the equation of state for an ideal gas

$$V = \frac{nRT}{P}$$

$$V_A = \frac{1.00(8.314)(600)}{25.0(1.013 \times 10^5)} = \boxed{1.97 \times 10^{-3} \text{ m}^3}$$

$$V_C = \frac{1.00(8.314)(400)}{1.013 \times 10^5} = \boxed{32.8 \times 10^{-3} \text{ m}^3}$$

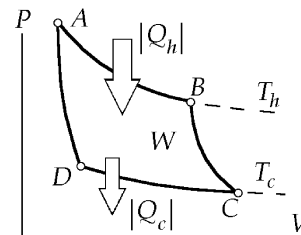


FIG. P22.70

Since AB is isothermal, $P_A V_A = P_B V_B$

and since BC is adiabatic, $P_B V_B^\gamma = P_C V_C^\gamma$

Combining these expressions,
$$V_B = \left[\left(\frac{P_C}{P_A} \right) \frac{V_C^\gamma}{V_A} \right]^{1/(\gamma-1)} = \left[\left(\frac{1.00}{25.0} \right) \frac{(32.8 \times 10^{-3} \text{ m}^3)^{1.40}}{1.97 \times 10^{-3} \text{ m}^3} \right]^{(1/0.400)}$$

$$V_B = \boxed{11.9 \times 10^{-3} \text{ m}^3}$$

Similarly,
$$V_D = \left[\left(\frac{P_A}{P_C} \right) \frac{V_A^\gamma}{V_C} \right]^{1/(\gamma-1)} = \left[\left(\frac{25.0}{1.00} \right) \frac{(1.97 \times 10^{-3} \text{ m}^3)^{1.40}}{32.8 \times 10^{-3} \text{ m}^3} \right]^{(1/0.400)}$$

or

$$V_D = \boxed{5.44 \times 10^{-3} \text{ m}^3}$$

Since AB is isothermal, $P_A V_A = P_B V_B$

and
$$P_B = P_A \left(\frac{V_A}{V_B} \right) = 25.0 \text{ atm} \left(\frac{1.97 \times 10^{-3} \text{ m}^3}{11.9 \times 10^{-3} \text{ m}^3} \right) = \boxed{4.14 \text{ atm}}$$

Also, CD is an isothermal and
$$P_D = P_C \left(\frac{V_C}{V_D} \right) = 1.00 \text{ atm} \left(\frac{32.8 \times 10^{-3} \text{ m}^3}{5.44 \times 10^{-3} \text{ m}^3} \right) = \boxed{6.03 \text{ atm}}$$

Solving part (c) before part (b):

(c) For this Carnot cycle,
$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{400 \text{ K}}{600 \text{ K}} = \boxed{0.333}$$

(b) Energy is added by heat to the gas during the process AB . For the isothermal process, $\Delta E_{\text{int}} = 0$.

and the first law gives
$$Q_{AB} = -W_{AB} = nRT_h \ln \left(\frac{V_B}{V_A} \right)$$

or
$$|Q_h| = Q_{AB} = 1.00 \text{ mol}(8.314 \text{ J/mol} \cdot \text{K})(600 \text{ K}) \ln \left(\frac{11.9}{1.97} \right) = 8.97 \text{ kJ}$$

Then, from
$$e = \frac{W_{\text{eng}}}{|Q_h|}$$

the net work done per cycle is
$$W_{\text{eng}} = e_c |Q_h| = 0.333(8.97 \text{ kJ}) = \boxed{2.99 \text{ kJ}}.$$

- P22.71 (a) 20.0°C
- (b) $\Delta S = mc \ln \frac{T_f}{T_1} + mc \ln \frac{T_f}{T_2} = 1.00 \text{ kg}(4.19 \text{ kJ/kg} \cdot \text{K}) \left[\ln \frac{T_f}{T_1} + \ln \frac{T_f}{T_2} \right] = (4.19 \text{ kJ/K}) \ln \left(\frac{293}{283} \cdot \frac{293}{303} \right)$
- (c) $\Delta S = +4.88 \text{ J/K}$
- (d) Yes . Entropy has increased.

ANSWERS TO EVEN PROBLEMS

- | | | | |
|--------|---|--------|---|
| P22.2 | (a) 667 J; (b) 467 J | P22.34 | (a), (b) see the solution;
(c) $Q_h = 149 \text{ J}$; $ Q_c = 65.0 \text{ J}$; $W_{\text{eng}} = 84.3 \text{ J}$;
(d) 56.5%; (e) $1.42 \times 10^3 \text{ rev/min}$ |
| P22.4 | (a) 30.0%; (b) 60.0% | P22.36 | 4.88 kJ/kg · K |
| P22.6 | 55.4% | P22.38 | (a) isobaric; (b) 402 kJ; (c) 1.20 kJ/K |
| P22.8 | 77.8 W | P22.40 | 3.27 J/K |
| P22.10 | (a) 869 MJ; (b) 330 MJ | P22.42 | 718 J/K |
| P22.12 | 197 kJ | P22.44 | (a) 39.4 L; (b) -2.50 kJ; (c) -2.50 kJ;
(d) -6.87 J/K; (e) +9.16 J/K |
| P22.14 | 546°C | P22.46 | 0.507 J/K |
| P22.16 | 33.0% | P22.48 | 34.6 J/K |
| P22.18 | (a) 5.12%; (b) 5.27 TJ/h;
(c) see the solution | P22.50 | (a) 2 heads and 2 tails;
(b) All heads or all tails;
(c) 2 heads and 2 tails |
| P22.20 | 453 K | P22.52 | 8.36 MJ/K |
| P22.22 | (a), (b) see the solution;
(c) 23.7%; see the solution | P22.54 | 32.9 kJ |
| P22.24 | 11.8 | P22.56 | see the solution |
| P22.26 | 1.17 J | P22.58 | (a) $2.62 \times 10^3 \text{ tons/d}$; (b) \$7.65 million/yr;
(c) $4.06 \times 10^4 \text{ kg/s}$ |
| P22.28 | (a) 204 W; (b) 2.43 kW | | |
| P22.30 | (a) 2.00; (b) 3.00; (c) 33.3% | | |
| P22.32 | (a) 51.2%; (b) 36.2% | | |

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P22.60 $\frac{\mathcal{P}T_c}{(T_h - T_c)c\Delta T}$

P22.62 (a) 4.11 kJ; (b) 14.2 kJ; (c) 10.1 kJ; (d) 28.9%

P22.64 see the solution

P22.66 $nC_p \ln 3$

P22.68 no; see the solution

P22.70 (a)

	P , atm	V , L
A	25.0	1.97
B	4.14	11.9
C	1.00	32.8
D	6.03	5.44

(b) 2.99 kJ; (c) 33.3%